Chapter 3
1, 2: What did we do?
3, 4: Let's learn some more!

Rute Borba – Universidade Federal de Pernambuco
Gilda Guimarães – Universidade Federal de Pernambuco
Edda Curi – Universidade Cruzeiro do Sul
Cristiano Muniz – Universidade de Brasília

In this chapter, we describe and analyse studies conducted by Brazilian researchers interested in investigating the processes of teaching and learning mathematics in the early years of schooling. To this end, we discuss the research carried out by Working Group 01, WG01, focused on the study of mathematics education in kindergarten and elementary school. We consider the interests of WG01 members in relation to the research themes chosen and the theoretical-methodological approaches adopted to investigate the complex phenomena of mathematical learning. The studies developed by this group, which involve students and teachers, have focused on an issue that still receives little attention, the early-education classes; on content introduced as part of recent curriculum changes in Brazil; and on public policies that aim to improve Brazilian mathematics education. The chapter also highlights the innovations associated with the research conducted by the group, as well as aspects that future research should consider. We hope to contribute to a better understanding on how teaching and learning of mathematics occur, and how it becomes a part of the early years of schooling in Brazil and other parts of the world.

3.1 Introduction

This chapter discusses trends in the research presented by Working Group 01 (WG01) on mathematics in kindergarten and elementary school education, which is a part of the Brazilian Society of Mathematics Education (SBEM – Sociedade Brasileira de Educação Matemática). Our analysis focuses on the studies presented in the VI International Seminar for Research in Mathematics Education (Seminário Internacional de Pesquisa em Educação Matemática – SIPEM), which was held in 2015. However, the chapter also considers studies presented in previous editions of the seminar, and other Brazilian studies in which researchers describe and analyse investigations in Brazil focused on the teaching and learning of mathematics in kindergarten and elementary school. We bring as reference works by

authors from recent national and international scientific events, such as the XII ENEM (Encontro Nacional de Educação Matemática) and the ICME 13 (International Congress on Mathematical Education), both of which occurred in 2016, to discuss and complement the production of WG01.

In Brazil, early childhood education corresponds to the age range zero - to five-year-olds, and to the 1st to 5th years of elementary school, intended for children from 6 to 10 years old. Children up to 3 years old attend nurseries and 4 and 5 year- olds attend kindergarten. Within these first years of schooling, it is important to also recognise the existence of the Youth and Adult Education mode (EJA – Educação de Jovens e Adultos), aimed at young people and adults who did not have access to schooling during their childhood years, experienced little success repeatedly, and/or dropped out of school.

The preparation of teachers for these levels of education is provided through teacher education in pedagogy courses, which usually include one or two disciplines related to the didactics of mathematics teaching, with a workload of only 60 to 80 hours each. This workload is not enough for deepening mathematical skills that should be developed to provide the future teacher with the necessary tools to teach this area of knowledge. It also means that opportunities to discuss curricula and the organization of mathematics teaching are limited. This is a strong indication that in-service education programmes are needed to complete the processes initiated in the pedagogy courses.

WG01was founded with the aim of discussing and disseminating research on teaching and learning of mathematics in kindergarten and elementary school, and organising them according to the following four mathematical areas described in the National Curriculum Parameters (BRASIL, 1997): *numbers and operations*, *magnitudes and measurements*, *space and shape* (geometry) and *data analysis* (statistics, probability and combinatorics). Since its inception, the research developed has focused, in general, on the following areas: a) children's and adults' knowledge of mathematical concepts in early schooling, developed inside and outside of the classroom; b) resources for teaching mathematics in kindergarten and elementary school; c) initial and in-service education of teachers of early childhood and elementary education and their practices; and d) the inclusion of students with and without special difficulties associated with learning mathematics in these years of schooling.

The second part of this chapter, through an analysis of research trends and a discussion regarding the specific objects investigated, brings the main objectives and the theoretical frameworks that support the studies, describes the research methods adopted and summarises the main results. It also situates the research concerning mathematics education of the early years of schooling developed in Brazil in the international arena. The studies are presented in categories to highlight the foci of the research works and the results Brazilian students present in mathematical learning during early-years education.

Thus, the title of the chapter is justified insofar as it indicates what researchers of WG01 have investigated, the main results of the research presented at SIPEM, the future trends of research that are needed to consolidate further understandings of learning at the beginning of schooling and, consequently, what teaching pro-

cesses might be the most effective. The chapter also considers contributions to initial and in-service teacher education programmes, and the practices of teachers who teach mathematics in kindergarten and elementary school.

3.2 Overview of WG01 investigations at SIPEM

From SIPEM I, held in 2000, to SIPEM V, held in 2012, seventy six studies were presented, thirty seven of which focused on the teaching and learning of mathematics in elementary school, thirty one focused on teacher education and 8 focused on mathematics teaching and learning resources in early schooling. This distribution shows that the research is more student-centred, although teachers and teaching and learning resources have also been targets of investigations.

During SIPEM I, WG01 concentrated exclusively on the first five years of elementary schooling, but, at the end of the event, it was decided that studies on kindergarten education (4-5 years old) would be incorporated into WG01 research, therefore, recognising that in kindergarten, prior to the five years of elementary schooling, mathematical learning occurs, and that this aspect ought to be part of WG01 investigations.

In line with this decision, since SIPEM II work focused on kindergarten education has been presented. One study (Selva 2003) discussed the use of bar charts, and investigated the feasibility of involving very young children in the study of statistical concepts, although there is little evidence that they are taught at this level of education. The results of this study align with the framework recently proposed by Kinnear and Clark (2016), stating that children from 5 years of age can already present statistical notions, such as the use of fundamental aspects of data handling and the use of their data observations to make decisions.

Elia and Mulligan (2016), in a retrospective study presented at ICME-13, state that, internationally, there has been an increase in research focused on early childhood education in recent years, although they also point to the wide variation among countries in the ages eligible for kindergarten and elementary school. Studies concerning the training of teachers for work in early childhood education have also increased internationally, but an issue that permeates the concern of national and international researchers is what kind of mathematics should be addressed in early childhood education. In some countries, such as Germany, according to Gasteigner and Benz (2016), it is not common to have explicit mathematics classes in kindergarten. This is also not usual in Brazil. In this sense, it is necessary to broaden the discussion about teaching at this level of education.

Studies focusing on the role of teacher education in teaching and learning mathematics in the early years of schooling have increased during the five editions of the SIPEM. Much of this research addressed mathematical content that teachers regard as difficult, such as rational numbers.

Research on textbook analysis was first presented at SIPEM III in 2006. These studies reinforced the interest of WG01 members in investigating resources for teaching and learning. Since SIPEM III the group has been concerned with these textbook-analysis studies because of the impact that this resource has on Brazilian education.

For the first time, at IV SIPEM, in 2009, WG01 was involved in discussing difference and inclusion. A study addressing the teaching of natural numbers to deaf children motivated the discussions (Nogueira & Silva, 2009). At the next SIPEM, the group debated the issue again. At the SIPEM VI, a WG specifically focused on inclusion and differences was established, and some of WG01 members chose to migrate to present studies in this new working group (WG13).

In the first five editions of SIPEM, different methodologies were used in the studies, including: participant observation, experimental research, document analysis, phenomenological approaches, didactic engineering and case studies, and a variety of theoretical frameworks and constructs informed the activities of data collection and analysis.

Considering jointly the research into the teaching and learning of kindergarten and elementary school-aged students, and the teaching and learning resources in early schooling, 30 out of the studies presented in the first five SIPEM were related to *numbers and operations*, 5 investigated *geometry* content, and 10 addressed statistics and combinatorics (*data analysis*).

In studies involving *numbers and operations*, the main references were Gérard Vergnaud, Raymond Duval, Jean Piaget, Delia Lerner and Patricia Sadovsky, whilst for the studies involving teacher education, Lee Shulman, Donald Schön, Deborah Ball and João Pedro da Ponte were often cited. These references have had an important influence on Brazilian studies, particularly in research carried out by members of WG01, and other researchers interested in the teaching and learning of mathematics in the early years of schooling. In this area, additive and multiplicative conceptual fields, including natural and rational numbers, were widely investigated.

Data analysis was also a content area that consistently attracted attention, with studies on statistical and combinatorial thinking. *Geometry* was studied more in the SIPEM I, and *magnitudes and measurements* were not the focus of research from editions SIPEM I to V. Other topics, such as mental calculation, assessment, inclusion and algebraic thinking, recognised as important issues in elementary education, were not addressed in depth within WG01, although these have been investigated within the Brazilian mathematics education community. An example is the recent research by Silva (2016), presented at the ENEM XII, who investigated the flexibility of mental calculation by children of the 4th year in multiplication and division operations.

The choice of themes, on the one hand, favoured teaching focuses, such as statistics, probability and combinatorics (i.e., *data analysis*), that have become part of the areas of mathematics in the early schooling years relatively recently, following the publication of the Parâmetros Curriculares Nacionais (BRASIL, 1997). On the other hand, there was little or no emphasis on other areas such as *geometry* and *magnitudes and measurements*. The trend to reduce interest in geometry seems to be global. As Clements and Sarama (2016) observe, this theme has played a limited role in mathematical research related to the early years of schooling and practice in the classroom, because of the lack of emphasis on curriculum and little teacher knowledge in this area. Regarding the lack of research on magnitudes and measurement, Chambris and Dougherty (2016) have noted that, besides our little

knowledge on concepts and skills related to measurement, in several countries the possible connections between magnitudes and measurement and other areas of mathematics are not being taught.

3.3 SIPEM VI: foci of research in mathematics education in the first years of early schooling

A diversity of research foci and methods were presented in the 15 studies that formed the basis of discussions in WG01 during SIPEM VI in 2015. This diversity can be observed in the tables below (Tables 1 and 2).

Table 1 shows mathematics areas and school levels of the 15 studies presented at the seminar, and four research foci are identified: students' learning of mathematical content inside and outside the classroom; teacher education; public policies related to teaching mathematics at the beginning of schooling; and the dialogic relationship between cognitive and emotional processes. Some of these issues had not been previously debated in WG01. In particular, it is interesting to note that the public policies implemented by the Brazilian government to improve the quality of teaching and learning in basic education were discussed in five papers, while one contribution considered a second new theme: students' emotional relations with mathematics and their learning of the discipline.

Regarding the foci of the research, students' learning and teacher education continue to be emphasised in WG01 investigations. This is not surprising, because investigations about students and teachers are always of interest to researchers in mathematics education, whether in Brazil or other countries.

The keen interest in public policies may have been motivated by the implementation and expansion in recent years of national and state government programmes for teacher education, the analysis of textbooks and large-scale assessments, which are aimed at teachers and students from the beginning of schooling. The analysis of public policies has become, in a very relevant way, the focus of investigations for researchers in mathematics education in Brazil, and this was particularly evident amongst the researchers who participated in WG01 during SIPEM VI.

A possible explanation for the limited number of studies focusing on emotional processes in WG01 may be the fact that researchers in this area choose to discuss their work in the working group designed specifically for the study of cognitive and linguistic processes (WG09).

In relation to the mathematical areas involved, Table 1 indicates that *numbers and operations* remains the mathematical topic that the members of WG01 most investigated, with seven studies directed at this area. Furthermore, in addition to natural numbers, there are studies on rational numbers, as both the additive and the multiplicative conceptual fields are investigated. Four studies on *geometry* and three on *data analysis* were also presented and, in this edition of SIPEM, three studies involved *magnitudes and measurements*. It is interesting to note that studies focusing on geometry previously addressed diagnostic studies of students and in the SIPEM VI involved documentary research and studies with teachers in

seeking to understand difficulties with teaching in that area. Studies that reference magnitudes and measurements directly addressed the knowledge of students and discussed public policies related to this area.

Table 1 – Foci of research in mathematics education in the early years of schooling in WG01

Focus of research	Author(s)	Area - (content)	School levels
Student learning	Azerêdo (2015)	Numbers and operations (multiplicative fields)	Elementary school
	Marques and Silva (2015)	Magnitudes and measures	Elementary school
	Scucuglia and Rodrigues (2015)	Numbers and operations (Digital Mathematical Performances)	Elementary school
	Silva and Borba (2015)	Data analysis (graphs and tables interpretation)	Elementary school
	Silva, Campos, Canova and Pinheiro (2015)	Numbers and operations (fractions in quotient situations)	Elementary school
	Tôrres and Muniz (2015)	Numbers and operations (numeric registration)	EJA (Youth and Adult Education)
Teacher education	Alencar (2015)	Numbers and operations (multiplicative fields)	Elementary school
	Guimarães and Oliveira (2015)	Data analysis (statistics - classification)	Kindergarten and Elementary school
	Vieira and Lobo da Costa (2015)	Geometry (use of technology)	Elementary school
Public policies	Bellemain (2015)	Magnitudes and measures (length)	Elementary school
	Curi (2015)	Numbers and operations and geometry (spatial relations and spatial geometric figures)	Elementary school
	Pacheco and Pires (2015)	Geometry (teacher's relations with curriculum material)	Elementary school
	Pinto Júnior, Tatagiba, Alarcão, Pereira and Souto (2015)	Numbers and operations, geometry, magnitudes and measures and data analysis	Elementary school
	Vieira and Nasser (2015)	No specific content	Elementary school
Cognitive and emotional processes	Medeiros (2015)	No specific content	Elementary school

It is noteworthy that the researchers of WG01 have turned their attention to different areas and mathematical content and contributed to discussions regarding teaching and learning different concepts of mathematics in early schooling. However, WG01 researchers have not studied intensively some specific content taught in the early years, such as algebra.

There may be a need for further investigation into the impact that algebra learning from the early school years can have on the development of more sophisticated algebraic thinking, as indicated by Kieran, Pang, Schifter and Ng (2016). Another path of research, influenced by Carraher and Schliemann (2015), is noted by Teixeira, Magina and Merlini (2016) who sought to investigate the understanding of algebraic structures in multiplication problem situations.

As for the levels of schooling addressed in WG01 research during the SIPEM VI, 14 studies were concerned with the regular initial years (from 1st to 5th grades of elementary school), either from the perspective of students or teachers working at this level. One study also addressed the kindergarten level and one focused on Youth and Adult Education (EJA). This was the first study regarding EJA presented in WG01 in the six editions of SIPEM.

As observed in relation to the previous SIPEM, there is a continuing need for studies concerning the kindergarten years, considering that mathematical learning starts early and it is possible, in appropriate ways (in a playful way, for example), to treat mathematical concepts with very young children. Further studies with teachers and students of the EJA (Youth and Adult Education) are also necessary, since this is a frequent teaching modality in Brazil, as in the country there are still many young people and adults with little schooling. To understand better how students with little schooling develop mathematical knowledge inside and outside the school, is, therefore, a very present challenge still in Brazilian reality.

3.4 SIPEM VI: theoretical and methodological bases of research in mathematics education in the early years

The 15 papers presented at SIPEM VI are shown in Table 2, highlighting the theoretical references that guided the studies and the methods used.

A variety of theoretical references supported the research. Some theoretical references are common to several studies; others are particular, in accordance with the subject of investigation. The references sometimes are specific to the field of mathematics education, and in other studies, the foundation draws from scholars in other areas such as psychology, sociology, language/linguistics and education.

Classical theories by scholars such as Jean Piaget referring, in particular, to the classification of collections of elements (Piaget & Inhelder, 1983) and with respect to geometry (Piaget & Inhelder, 1993), were used. Other classical theories referenced came from the work by Lev Vygotsky, discussing the historical-cultural perspective of learning (Vygotsky, 2007), and the assumptions of Leontiev (2004) about the process of involvement in activities. The impact of classical theories (constructivist, socio-interactional and historical-cultural) is, thus, evident in mathematics education of the early years of schooling.

Table 2 - Foci, authors, titles, theoretical and methodological references of the research presented at the SIPEM VI

Focus	Author(s)	Title	Theoretical Reference(s)	Method
Student learning	Azerêdo (2015)	Registros semióticos do campo multiplicativo: um instrumento para o ensino nos anos iniciais	Duval (2009, 2011); Nunes and Bryant (1997); Vergnaud (2009)	Intervention study (teaching)
	Marques and Silva (2015)	Alfabetização na perspectiva da racionalidade aberta	Galvão and Di Pierro (2007); Pérez (2008)	Diagnostic study
	Silva and Borba (2015)	Desempenho de estudantes em itens sobre tratamento da informação na Provinha Brasil de Matemática	Guimarães and Oliveira (2011)	Documental research and diagnostic study
	Silva, Campos, Canova and Pinheiro (2015)	Equivalência em situação quociente: uma análise de dois estudos	Nunes and Bryant (1997, 2009); Streefland (1984, 1997)	Intervention study (teaching)
	Scucuglia and Rodrigues (2015)	A produção de performances matemáticas digitais nos anos iniciais do ensino	Borba and Villarreal (2005); Gardner (1993)	Intervention (teaching)
	Tôrres and Muniz (2015)	Identificação e análise de conhecimentos numéricos de jovens e adultos, em explicações orais e escritas	Brousseau (2008); Freire (1987); Schoenfeld (1992); Vergnaud (1990)	Diagnostic study
	Alencar (2015)	Os referenciais teóricos norteadores de pesquisas sobre a formação contínua de professores dos anos iniciais no campo conceitual multiplicativo	Shulman (1986); Vergnaud (1990)	Literature review
Teacher education	Guimarães and Oliveira (2015)	Relação entre saber classificar e o domínio da linguagem oral como determinantes na explicação oral de professores dos anos iniciais	Oritz (1994); Piaget and Inhelder (1983); Proulx, Bednarz and Kieran (2006); Schneuwly and Dolz (1997)	Diagnostic study

Continued - Table 2 - Foci, authors, titles, theoretical and methodological references of the research presented at the SIPEM VI				
Focus	**Author(s)**	**Title**	**Theoretical Reference(s)**	**Method**
Teacher education	Vieira and Lobo da Costa (2015)	Perspectivas de mudanças no ensino de geometria com o uso de tecnologia digital: experiências em grupo de estudos de professores dos anos iniciais	Leontiev (2004); Mishra and Koehler (2006); Murphy and Lick (1998); Shulman (1986)	Action-research
Public policy	Bellemain (2015)	Ensinar comprimento no ciclo de alfabetização	Chevallard (1985, 2002); Douady and Perrin-Glorian (1989)	Documental research
	Curi (2015)	Orientações curriculares, livros didáticos, Prova Brasil de matemática do 5º ano e práticas de sala de aula: resultados de uma pesquisa longitudinal	Lerner and Sadovski (1996); Piaget and Inhelder (1993); Sacristán (2000); Van Hiele (1986); Vergnaud (1996)	Documental research
	Pacheco and Pires (2015)	Espaço e forma nos anos iniciais e o uso de materiais curriculares	Brown (2009); Crowley (1994); Parzysz (1988); Piaget and Inhelder (1993)	Documental research and diagnostic study
	Pinto Jr, Tatagiba, Alarcão, Pereira and Souto (2015)	Matrizes de Matemática do Saeb: um estudo comparativo entre 3º e 5º anos do Ensino Fundamental	Neidorf et al (2006)	Documental research
	Vieira and Nasser (2015)	PNAIC no Estado do Rio de Janeiro: investigando as práticas dos formadores numa perspectiva interdisciplinar	Fazenda (1992, 2003); Japiassú (1976); Santomé (1998)	Case study
Cognitive and emotional processes	Medeiros (2015)	Satisfação e Conhecimento Matemático: uma pesquisa sobre afetos com alunos dos anos iniciais do ensino fundamental em uma escola pública do Distrito Federal, Brasil	Gómez Chacón (2003); González Rey (2005); Morin (2011); Vygotski (2007)	Participant observation

In SIPEM VI, Gérard Vergnaud, with his theory of conceptual fields, continues as a reference in many of the studies: with regard to conceptualisation (Vergnaud; 1990) and the classification of additive and multiplicative problems (Vergnaud;

1996, 2009). Other references of a psychological nature include the studies of Terezinha Nunes and Peter Bryant, in particular, in the classification of multiplicative problems and in the discussion of equivalence in quotient situation (Nunes & Bryant 1997, 2009). One of the studies referred to multiple intelligences (Gardner; 1993). In this sense, the strong influence of psychology in the Brazilian studies of early school years is observed, especially in the consideration of the cognitive elements of mathematical conceptualisation.

From a psychological and linguistic perspective, Raymond Duval also continues to be a reference for studies on the role of semiotic representations in students' learning of mathematics (Duval 2009, 2011). Other research outside SIPEM also uses the theory of register of semiotic representation as a reference in studies of the early years of schooling. Such research includes, for example, investigation by Borba, Azevêdo and Bittar (2016), presented at ICME 13, who analysed the contribution in textbooks of various symbolic representations and conversions between them in the development of combinatorial reasoning.

Other approaches based on linguistic studies, such as the work by Schneuwly and Dolz (1997), are referenced with respect to verbal and nonverbal elements of spoken language, as is Oritz's work (1994) on the need for communicative competence of the teacher; and Proulx, Bednarz and Kieran (2006) are referenced with respect to teachers' explanations as a regulatory function of learning and of students' development of knowledge.

Scholars focused on mathematics didactics were also used as the basis of WG01 studies discussed during SIPEM VI. Guy Brousseau and his theory of didactic situations served as a reference regarding the notion of the didactic contract (Brousseau 2008). The anthropological theory of the didactic (Chevallard 1985, 2002) and the argument of Douady and Perrin-Glorian (1989) on the need to articulate three domains (i.e., geometric, of magnitudes and of measurements) were the theoretical basis of a study on magnitudes and measurements. The teaching approaches of Lerner and Sadovski (1996) and Saiz (2006) were also referenced. These cited references indicate the importance of the didactics of mathematics in WG01 studies.

From the area of educational research, Lee Shulman and his position regarding teachers' content and pedagogical knowledge (Shulman, 1986) continued to be a theoretical reference used in the studies considering the practices of teacher, as was the case in previous editions of SIPEM. The work of Mishra and Koehler (2006) was used as a theoretical basis concerning technological content knowledge. Some studies cite Paulo Freire, in particular as regards to the recovery of extra-curricular knowledge (Freire 1987); Gimeno Sacristan, as regards to curricula (Sacristan 2000); Fazenda (1992, 2003), Japiassú (1976) and Santomé (1998), who are cited with regard to interdisciplinary learning; Murphy and Lick (1998), to support the creation of a study group; Morin (2012), considering knowledge as a complex construct; Almeida (2010), discussing learning by culture in line with school knowledge; Pérez (2008), comprising literacy as a resignified production; and illiteracy concepts mentioned by Galvão and Di Pierro (2007). Neidorf et al. (2006) are referenced regarding the analysis of testing. It is observed

that the breadth of educational theorists used in SIPEM involves researchers not specifically concerned with the learning of mathematical knowledge.

Many mathematics educators were also used as references for the studies in SIPEM VI. Alan Schoenfeld's work is cited as related to metacognition processes (Schoenfeld 1992) and Tony Brown's studies of teachers' relations with curriculum materials is also mentioned (Brown 2009). Borba and Villarreal (2005) were used in a study with the concept of *human-with-media* interaction, and, geared to specific mathematical content; other authors were referenced concerning the teaching and learning of rational numbers (Streefland 1984, 1997) and the theorising of geometric thought (Van Hiele 1986; Parzysz 1988). The discussions of feelings and emotions in mathematical learning referred to Gómez Chacón (2003), González Rey (2005) and Morin (2011).

In short, the various frameworks of WG01 studies presented at SIPEM VI tend to have their roots in classical theories (i.e., constructivist, socio interactionist and historical-cultural). These studies are strongly influenced by psychology, specifically concerning the relations with cognition and, in particular, the analysis of the role of symbolic representations in the teaching and learning of mathematics. They are also influenced by the didactics of mathematics and studies of education of a broader nature. We see, therefore, these diverse references pointing to the variety of perspectives and approaches that are possible in research in mathematics education in the early years of schooling.

Table 2 also indicates a great variety of methods used in the research discussed in WG01 relating to the early years of schooling. This collective of studies shows that there is a need for more intervention studies (i.e., teaching children enrolled in kindergarten and elementary school).

3.5 Main results of the studies of WG01 at SIPEM VI

3 5.1 Elementary-school students' learning

With respect to this research focus, six studies were presented and discussed in WG01 during the SIPEM VI, including three diagnostic studies with data from tests and observations (one involving documental research and diagnostic study) and three intervention studies (i.e., research that involves teaching). Five of the studies were conducted with elementary-school children and one with EJA students. There was clearly great interest amongst WG01 researchers in conducting diagnostic studies directly with children, young people and adults.

In the sixth edition of SIPEM, like in the previous editions, the most frequently investigated mathematical area was *numbers and operations*, but there was also interest in the study of *data analysis*. Investigations addressed most recently included curriculum content, such as interpreting graphs and tables, and difficult content, such as multiplicative structures.

One of the studies (Azerêdo; 2015) aimed to understand the role of treatment and conversion of semiotic registers (as discussed by Duval; 2009, 2011) of 105 3rd, 4th and 5th grade students in activities involving multiplication and division, in proportion and distribution problems and a conversion activity in the direction text-drawing. The results signalled considerable difficulty in solving problems and the usage of a variety of semiotic registers, with the use of formal algorithm evidenced only by 5th grade students. When the children converted text into drawing, the results indicated an increase in the success rate, which points to the mediation role played by drawings.

Another study referred to school and extracurricular knowledge of students based on narratives and observations (Marques & Silva, 2015). The research consisted of a study of the relations between mathematical knowledge developed within and outside school, discussing cultural learning, as indicated by Almeida (2010). The study was conducted with 13 children aged 7 and 8 years old in a multigrade[1] class at a riverside community in the Amazon. For data collection, classes were observed, as were recreational activities and the journeys between school and children's homes on the school boat. Most of the time, children related *learning* only to the classroom, not recognising their learning beyond the school environment. When reporting their experiences of the harvesting and marketing of *açaí* (a fruit from northern Brazil), children showed knowledge of magnitudes and measurements, when comparing *açaí* tree heights and when demonstrating their knowledge of the supply-and-demand market system using the example of the price variation of a 'rasa' (a measure of capacity) of *açaí.*

This study verifies how important it is to have more studies focused on mathematical knowledge that children and adults develop in ways that go beyond what is learnt in school, reinforcing the well-known work developed by Carraher, Carraher and Schliemann (1985). WG01 researchers recognise strongly that much mathematics is learned out of school and discuss the role of school in mathematical development.

A third diagnostic study (Silva & Borba, 2015) regarded children´s data-analysis in *Provinha Brasil*, also known as Children's Literacy Assessment. It is a diagnostic evaluation that aims to investigate the development of literacy skills of children enrolled in the 2nd year of elementary education in Brazilian state schools. *Provinha Brasil* is applied twice a year (at the beginning and the end of a school year) to have a more accurate diagnosis of children's learning in reading skills and mathematics. Adherence to this assessment is optional, and the application is at the discretion of each education department of the Brazilian states, with the implementation date of *Provinha Brasil* following a decision of each state´s school system.

In addition to observing the performance of 40 children from a state school of Rio Grande do Sul in each of the 18 data-analysis items of various editions of the

[1] Multigrade classes are common in Brazil, especially in rural areas where there is a shortage of teachers, students or resources. In these classes, students of different ages and levels of schooling are taught by the same teacher.

Provinha Brasil, the researchers analysed documents from the National Institute of Studies and Research Anísio Teixeira (INEP)[2]. The authors problematise how national evaluation questions can relate to the performance of children in interpreting graphs and tables items. The results indicate that the mode of presenting the items, the contexts involved and the numerical values influence the success and the children's interpretation ability, corroborating with the studies of Guimarães and Oliveira (2011). Usual beliefs of many mathematics educators, such as improved performance in evaluation due to the use of images or that familiar contexts facilitate reasoning, however, were not confirmed in the study. Regarding large-scale assessments and items related to graphs and tables in *Provinha Brasil*, the findings suggest that they should cover wider items and not only those familiar to children, since the images shown in statements or familiar with the subject does not always favour a proper assessment of children´s performance. A wider cover of items can enable children to understand the functionality of graphs as a tool to communicate data, as indicated by Leavy and Sloane (2016).

Another investigation with students was presented with a focus on multiplicative structures (Silva, Campos, Canova & Pinheiro 2015), particularly on how students learn about equivalence between fractions in quotient situations. Research by Nunes and Bryant (1997, 2014) and Streefland (1984, 1997) were used as theoretical frameworks for discussing the teaching and learning of rational numbers. Two studies were carried out: one with 163 5th-grade students and another with 20 students aged between 10 and 11 years old, who responded to diagnostic instruments and learnt about quotient situations. It was found that, initially, a very low percentage of students (less than 2%) showed understanding and could justify the equivalence between fractions. However, it was observed that some students, after exploring the quotient meaning, presented correct representations and understandings of equivalence between fractions. Nunes (2016), in a research review presented at ICME 13, defends that quotient situations bring opportunities for the students to relate with everyday situations, to use quantitative reasoning and to use representational tools that help them mathematise situations. It was noted that understanding the meaning of fractions can be extended if, during teaching, teachers use ratios to help students make sense of the symbols and how to manipulate them. Similar results were obtained by Utimura and Curi (2016) in a study, presented at ENEM XII, with 4th grade teachers and students learning rational numbers. The authors noted that meanings underlying representations came to be understood by students through teacher mediations and student interactions.

In another study with children (Scucuglia & Rodrigues, 2015), the role that music and film can play in mathematical learning was examined in the context of digital mathematics performances (DMP). The study took as one of its theoretical

[2] The National Institute of Educational Studies Anísio Teixeira (Instituto Nacional de Estudos e Pesquisas Educacionais - INEP) is under the Ministry of Education (MEC) and aims to support the formulation and implementation of public policies for the area of education in Brazil, promoting studies, research and periodic reviews of the Brazilian educational system.

bases the notion of *human-with-media* interaction, as discussed by Borba and Villarreal (2005). In the study, teaching sessions were conducted with four groups of students (47 in total) in which they produced digital mathematical performances from video recordings. One way to get students to reflect from recordings can be by producing DMPs, thereby relating meaning, emotion and mathematical communication. However, besides good artistic quality (visual and musical, for example), in the production of a DMP it is necessary for the mathematical meanings involved to be fully met, that is, mathematics should be kept in first plan.

A single study with youth and adult students was presented at the SIPEM VI at WG01 (Tôrres & Muniz, 2015). This study aimed to identify and analyse the production of mathematical knowledge of young people and adults in varying degrees of oral and written formalisation, of cognitive and meta-cognitive knowledge of the decimal numerical system, focusing on the impossibility of determining accurately the existing boundaries between students' school and extracurricular math skills. Just as with children, it was observed that the production of school mathematical skills by young people and by adults was affected by the locus and didactic contract (from the perspective of Brousseau, 2008) established between educator and students. Even outside of the classroom, it was observed that the rules of the didactic contract are present in the minds of students and are revealed as they perform mathematical tasks, even in the absence of the teacher. Also, students explain what they know or do not know using self-regulation arising from their beliefs and intuitions about mathematics.

3 5.2 The training of teachers who teach mathematics in elementary education

Focus on teacher training was a trend observed over the six editions of SIPEM. Nearly 40% of the research studies presented in WG01 focused on this theme. This corroborates the evidence presented by Lee and Lin (2016) indicating the ever-growing volume of international research in recent years regarding the education of elementary school teachers. However, contrary to the international trend of the privileging of studies with teachers in initial teacher education courses, WG01 has contemplated studies almost equally about initial and continuing teacher training.

As shown in Tables 1 and 2, three of the studies presented in 2015 focused on teacher training. All three studies involved the elementary school level; the first study was a literature review, the second was a diagnostic study and the third was an action-research project. The absence of studies with kindergarten children or EJA students indicates a need for research directed towards these areas, because they have specific characteristics that are paramount for the training of elementary school teachers.

The topics that were treated in these studies were, once again, *numbers and operations* (more specifically, multiplicative structures) and *data analysis* (regarding element classification), which would indicate a greater concern with teacher training in the most recently inserted area in the Brazilian curriculum (i.e., data analy-

sis) and contents that are considered difficult (such as multiplication, division and rational numbers). There is a consensus around the idea that they are difficult topics, in particular, how difficult it is to understand rational numbers and, especially, how to teach this topic, which has been discussed in several studies, as noted by Tirosh (2000).

The first of these studies (Alencar, 2015) presents literature review of theoretical frameworks that guide research toward continuing education of early-school teachers on the multiplicative conceptual field (as described by Vergnaud, 1990). This literature review was based on the databank of dissertations and theses of CAPES[3]. It was found that the focus of the investigations centres on theoretical issues (Shulman, 1986, among others), discussing teacher knowledge, teacher professional development, and reflective practice essential for an effective teacher training that enables mathematical developments by elementary school students.

The second project, a diagnostic study, involved future teachers (Guimarães & Oliveira, 2015) and aimed to investigate the construction of criteria for conducting classifications and students' oral explanations about the categorisations made by colleagues, verifying the influence of initial education in the ability to classify and explain categorisations made. To do this, classification activities by 113 students from three different nationalities (Brazilian, Canadian and Spanish) were observed. Analyses of the results indicated that the participants had difficulties in creating classification criteria regarding *exhaustiveness* and *exclusiveness*. Other studies also observed difficulties with classification (Kinnear & Clark, 2016; Estrella & Olfos 2016), showing that classification is not an act as simple as sometimes thought. Learning to collect and organise data is a fruitful activity for students from early childhood education so that they can develop statistical reasoning (Kafoussi; 2016; Vidal-Szabó, Estrella & Morales, 2016). Added to this, an oral explanation in mathematics can facilitate learning by linking the domain of oral discourse and the mathematical concept domain. The domain of only one of these aspects is insufficient for ensuring a quality explanation. It is essential that teachers be aware that their oral discourse may provide a greater or lesser quality of student learning and that it can lack a domain and articulation between ordinary language, mathematics language and the mathematical concepts.

Another study (Vieira & Lobo da Costa; 2015) sought to analyse processes associated with the appropriation of digital technology in a study group composed of elementary school teachers, through the use of software using 2- and 3-dimensional geometric figures. This was an action research project aimed at encouraging teachers to mobilise technological pedagogical knowledge, technological knowledge of content (as indicated by Mishra & Koehler, 2006), and specific knowledge content in the discussion of geometry concepts. It was observed that

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3 The Coordination of Higher Education Personnel Training (Coordenação de Aperfeiçoamento de Pessoal de Ensino Superior - CAPES) is a foundation linked to Brazil's Ministry of Education (MEC) and is engaged in the expansion and consolidation of post-graduate studies (i.e., for Masters and Doctorate degree programs) in all Brazilian states.

the teachers understand the nature of the possibilities and limitations of software applications and envision how these are related to utility in their teaching practices.

3 5.3 Relations of public policies with mathematics in the first years of elementary school

In SIPEM VI, one of the strongest research focus was on public policy, with five studies addressing elementary schooling public policies. Again, research on kindergarten and to adult education were not presented and public policies aimed at younger children or young people and adults in early schooling were not discussed.

Three of the studies involved documentary research, one articulated documentary research with a diagnostic study of teachers and students and the other was a case study on the National Pact for Literacy at the Right Age (Pacto Nacional pela Alfabetização na Idade Certa - PNAIC), described below. The four areas specified in the National Curriculum Parameters (PCN) (i.e., *numbers and operations*, *geometry*, *magnitudes and measurements* and *data analysis*) were involved in the studies, highlighting the breadth of research focused on public policy, using varied content and mathematical concepts from the analysis of documents and observations of teachers and students.

The first of these studies (Bellemain, 2015) aimed to analyse nationwide curriculum guidance documents and books approved by the public policy textbook evaluation programme (Programa Nacional do Livro Didático—PNLD) for the first three years of schooling (literacy cycle), regarding the teaching of concepts related to length.

The National Textbook Programme (PNLD) aims to subsidise the pedagogical work of teachers by distributing collections of textbooks to students of basic education. These collections are evaluated by researchers and teachers of different educational levels and described in a guide for teachers. Each year, the Ministry of Education of Brazil (MEC) acquires and distributes books contained in the guides to all students at a school level from elementary, middle and high school education.

In this first study related to public policies, relations were observed between what is prescribed in the curriculum guidance documents and the curriculum presented in textbooks. It found a positive variety of types of activities proposed in the books, but the emphasis on teaching length lies in the use of units and conventional instruments, which differs from what is preconised in the curriculum guidelines. It was also noted that content treating aspects of length is strongly focused on strengthening numbers and numerical operations (contrary to the recommendations by Douady and Perrin-Glorian 1989, on the need to articulate geometric, magnitude and the measurement domains) and focusing on numbers and operations bring little benefit to teachers on how to explore connections with extracurricular practices.

The importance of investigations of educational resources such as textbooks is emphasised and has been a constant concern for WG01 members. Research in this area has pointed to improvements in the quality of textbooks in Brazil and indicates further work that still is needed. Public policies related to these improvements have been in place in Brazil since the mid-1990s, in contrast to other countries, in which government actions in this direction have been implemented only recently. In Ireland, for example, it was only in 2011 that the National Council for Curriculum and Assessment decided to intervene in the development of textbooks (O'Keeffe, 2014), and in other countries, such as the Czech Republic, according to Moraová (2014), many books that are currently used were published in the early 1990s and have become outdated over time.

The second study regarding the focus upon public policies (Curi, 2015) aimed to analyse curriculum guidelines, textbooks and external evaluations that public policy bodies have developed to assist teachers in their mathematics teaching practices in the classroom. The study was motivated by the difficulties teachers experience in accessing and understanding official documents, in particular with regard to *numbers and operations* and *geometry*. It involved an analysis of curriculum documents from the Secretary of Education of São Paulo that report on results of the SAEB (an evaluation system described below), collections of textbooks, testimonials and teacher reports.

The System of Basic Education Assessment (SAEB - Sistema de Avaliação da Educação Básica) consists of two processes: The National Basic Education Assessment (ANEB - Avaliação Nacional da Educação Básica) and the National School Performance Assessment (ANRESC - Avaliação Nacional do Rendimento Escolar). ANEB is carried out by sampling in each Brazilian state and focuses on the efforts of educational systems, and receives the name SAEB in its dissemination. ANRESC is more extensive and detailed than ANEB and focuses on each Brazilian school, receiving the name *Prova Brasil* in its dissemination.

In this second study, it is evident that there is consistency between the prescribed curriculum documents and the assessment instruments proposed by public bodies and aimed at improving the teaching of mathematics. However, in the *presented curriculum* (textbooks) and the *curriculum-in-action* (experienced in the classroom), using the terminology proposed by Sacristan (2000), there are major differences between the *prescribed* and the *evaluated curricula*. In the *presented curriculum*, gaps were observed, for example, in addressing characteristics of the decimal number system (as also indicated by Lerner and Sadovski 1996), an absence of some of the meanings associated with numerical operations, and, in relation to geometry, books do not contain different representations of space - contrary to recommendations made by Saiz (2006) and championed by Nascimento and Fernandes (2016). Moreover, the greatest difficulties are the actions of the teachers, who, despite presenting planning objectives consistent with the curriculum documents, reveal a practice that resembles their experiences as students of basic education.

The third study that focused on public policies (Pacheco & Pires, 2015) problematised the fact that education departments of different Brazilian regions have produced and implemented curriculum materials without proper monitoring and

without drawing on the results of scientific research. The objective was, therefore, to analyse how these materials are interpreted and used in the classroom by teachers in the early school years. The study focused specifically on material related to geometry. An analysis was made of the curriculum materials associated with the project "Mathematics Education in the Early Years", in the light of the model of the teacher's relations with curriculum material, proposed by Brown (2009). Observations of two teachers' classes in which the material was used (in the 1st and 5th year of elementary education in state schools in São Paulo) were undertaken and semi-structured interviews with teachers were also carried out. It was noted that curricular materials can have greater impact on teaching practice than the prescribed curriculum. It was also observed that teachers use the materials in different ways (reproduce, adapt or create situations), reflecting their views and knowledge. Similarly, Cortês and Muniz (2016) analysed implicit curriculum concepts in the practices of early schooling teachers and observed that public policy teacher education has partially influenced the practices; the prescribed curriculum has had little reach in practice, and learning assessments usually occur at the end of the process, making curricular reorganisations impossible.

The fourth study focused on public policies (Pinto Junior, Tatagiba, Alarcão, Pereira & Souto, 2015) observed convergences and divergences between 3rd and 5th grade reference matrices of the SAEB described above. The analysis of documents in which the results associated with SAEB are presented (Brasil, 1997, 2002, 2012, 2013) demonstrated that the assessments present convergences between 3rd and 5th grade reference matrices, both in their dimensions of content and cognitive dimension, although the 3rd grade reference was constructed 12 years after the 5th grade reference, and by another group of experts. Advances in skills that are measured by the national assessment system were also observed, except in the area of *data analysis*, which is probably justified by more recent discussions in skills to be developed in statistics education.

The fifth study, focused on the analysis of public policies (Vieira & Nasser, 2015), is a case study that aimed to analyse the challenge of developing an interdisciplinary perspective upon mathematics and Portuguese in the teacher education programme, National Pact for Literacy at the Right Age (PNAIC).

PNAIC is a national public policy that aimed at ensuring a high quality of literacy education during the first three years of schooling. This programme, conducted in all Brazilian states in 2014, led experts in the areas of language and mathematics to develop and implement interdisciplinary in-service education processes.

This fifth study applied questionnaires with 14 Portuguese and 13 mathematics PNAIC group members of the state of Rio de Janeiro. The results indicated the need for breaking paradigms, such as the teaching of isolated disciplines (see Fazenda 1992, 2003; Japiassú 1976; and Santomé, 1998, who point towards an interdisciplinary search for completeness, but considering also the particularities of the disciplines) deepening concepts; valuing collective decisions and exchange of knowledge between areas; providing different perspectives and more productive discussions. In a similar direction, Souza, Souza and Passos (2016) found contributions of the language area, in children's stories suggested in PNAIC note-

books for the mathematical literacy of children of the first three years of schooling.

3 5.4 Dialogues between cognition and emotion in mathematics learning

One study presented at SIPEM VI (Medeiros, 2015) focused on affection and the satisfaction of students in the early years of elementary education concerning mathematical knowledge, emphasising that mathematics learning involves cognition and affection (Gómez Chacón, 2003). The paper represents an area little explored in WG01. The investigation was characterised as participant observation within a 4th-grade class (children 8-10 years) in a state school. The study noted that for a better acquisition of mathematical knowledge by the students, it is also necessary to consider the satisfaction of students' learning processes. Similarly, in another study (Muniz, 2016) presented at ENEM XII, a WG01 researcher sought to understand the production of subjective senses in the history of mathematics learning of children at risk, i.e., those that have their fundamental rights violated or threatened by injury, whether by act or omission of the society or the state; or missing, omission or abuse of parents or guardians. The study enabled a schemes analysis and revealed elements of the complex learning processes of these children.

3.6 Final thoughts

This chapter has aimed to show the approaches adopted in WG01 to investigate the development of mathematical knowledge of students in and out of school, initial and continuing teacher education, a variety of resources for the teaching of mathematics, and an inclusive perspective of mathematics education. Some topics have been intensely investigated, such as the mathematical conceptual development of children and teacher training. Others have been the subject of a good number of investigations, such as teaching resources (textbooks and new technologies). Some topics require further research and debate, especially kindergarten education, teaching and learning of young people and adults in early education, the development of mathematical knowledge in extracurricular environments and the inclusion of all children in mathematical learning. Some of the topics have been the research targets of other working groups of the Brazilian Society of Mathematics Education, nonetheless, it is considered important that they are also targets of specific discussions by the research group aimed at studies on teaching and learning of mathematics in the first years of schooling.

The body of studies developed by WG01 has involved the following different mathematical areas: most of the studies related to *numbers and operations*; a good

number of studies focused on *geometry* and *statistics* and *combinatorics* (but no study focused on *probability*), and a few studies addressed magnitudes and measurements. Other Brazilian studies regarding probability in the early years (such as Batista & Borba, 2016) have been conducted, although these studies have not been presented at WG01. Therefore, the debate regarding different concepts and mathematical areas are valued by Brazilian researchers of the initial years of schooling, in order to provide a balanced weight to diverse mathematical knowledge in the classroom.

Different problematisations have generated distinct themes and diverse objects of investigations, with different research goals from diverse theoretical frameworks, which are largely established in the international literature. These references are from the area of mathematics education, in conjunction with other areas of knowledge, such as education, psychology, linguistics and sociology.

In WG01 studies, several investigative methods were used (mostly diagnostic and very few intervention projects): case studies, participant observation and documental research. This variety of perspectives may contribute to better investigate processes associated with teaching and learning, and enable both directly and indirectly the understanding of practices and uses of mathematics. It is noteworthy, however, that few of the investigations aimed to analyse specifically teaching and learning in the classroom, and this is one of the focuses that needs more research.

Several specific concepts have been the focus of WG01 investigation with emphasis on contents that are considered difficult, both in the understanding of children and of teachers. For example, *multiplicative structures* have been the subject of research from different perspectives - the student, the teacher, the curriculum and teaching resources. More recent topics of the curriculum proposals, such as statistics and combinatorics, have also been the subject of constant research. Algebraic thinking, however, is a topic that still needs to be the focus of research, since recent curriculum proposals have noted the need to work with algebraic concepts from the beginning of schooling. This should boost investigations into practices in the classroom and teacher education to work on the development of algebraic thinking.

The focus of research in recent areas of national and state curricula and in some contents rarely worked in class seem to have been some of the strongest contributions of WG01 of the Brazilian Society of Mathematics Education, including: a) the investigation on how school students can develop different modes of reasoning, such as geometric, statistical and combinatorial, among other ways of thinking, and b) establishing the possibility of treating more complex concepts with children from kindergarten and elementary school.

Among the consensuses resulting from the various studies conducted by WG01 is a vision of mathematics articulated with other areas of education and supported in meaningful contexts for students, including daily extracurricular activities. Beliefs, emotions and desires are also aspects to be considered.

Also, in many studies conducted by the researchers of the group, it is recognised that symbolic representations play a very important role in mathematics, and in explanation, argumentation and communication in teaching and learning processes. These are essential tools for mathematical knowledge and are not only ac-

cessories to this understanding and, therefore, require close attention in the monitoring of teaching and learning processes, mostly in the first years of schooling.

In particular, regarding government action, it is noteworthy that several WG01 studies examine public policy regarding teacher training, courseware production and large-scale assessment of students' mathematical performances. It is necessary to investigate further teachers' understanding in relation to these policies and their implementation in the teaching of mathematics, to understand better how the prescribed curriculum, based on research, can be approached in the classroom. Assessment processes also require special attention to examine the knowledge acquired and in development by the students; and serve to support the work of teachers in the classroom, allowing reflections and improvements in the quality of mathematics teaching in early school years. Thus, WG01 studies on the learning of students have sought to allow teachers a better understanding of the processes of teaching and learning, including the evaluation processes conducted locally and those that result from public large-scale evaluation policies.

The investigations of Brazilian public policies, such as the evaluation of textbooks, continuing teachers' education and assessment of student learning, among others, point to some of the advances that these policies have provided for Brazilian mathematics education, preparing teachers for practice in the classroom, providing high quality educational resources and checking advances in the mathematical knowledge of students in early schooling. They also show that there is still much to be done.

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